Vibrations of Orthotropic Parallelogramic Plates with Variable Thickness

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The vibration problem of thin orthotropic skew plates of linearly varying thickness is analyzed under the assumptions of small-deflection theory of plates. Using dimensionless oblique coordinates, the deflection surface is expressed as a polynomial series satisfying the required polar symmetry and the boundary conditions. For plates clamped on all the four edges, numerical results for the first two natural frequencies are presented for various combinations of aspect ratio, skew angle and taper parameter. Convergence study has been made for a typical configuration of the plate and the limited available data is inserted therein along with the computed results, for comparison.

Nomenclature

$2a$, $2b$	= dimensions of plate
A_{mn}	= coefficient of series representing deflection
$B_{mn}^{(ik)}$	= element of mass matrix
$C_1, C_2, C_3, C_4, \ldots, C_{12}$	= constants in Eq. (15)
$D_{mn}^{(ik)}$	= element of stiffness matrix
D_x, D_y, D_{xy}	= variable rigidity parameters of orthotropic plate
D_{x_0}	$= E_x h_0^3/12(1-\nu_{xy}\nu_{yx})$
D_{y_0}	$= E_y h_0^3/12(1-\nu_{xy}\nu_{yx})$ = rigidity constants of orthotropic plate of uniform thickness
$D_{x_0y_0}$	$= G_{xy} h_0^3/12$
D_1	= cross-rigidity of orthotropic plate
E_x, E_y, G_{xy}	= material constants of orthotropic plate
h	= thickness of plate with taper
h_0	= thickness of plate without taper
i, k, m, n	= positive integers
M_x, M_y, M_{xy}	= moment resultants
p_0, m_0	= parameters relating rigidity constants
Q_x, Q_y	= transverse shears in x and y directions, respectively
t	= time
w	= deflection of plate independent of time
W	= lateral deflection of vibrating plate
x, y	= Cartesian rectangular coordinates
$\bar{x}, \bar{y}$	= dimensionless oblique coordinates
α	= taper parameter
γ	= a/b, aspect ratio of plate
θ	= skew angle
ν	= Poisson's ratio of isotropic material
ν_{xy}, ν_{yx}	= Poisson's ratio in orthogonal directions
ξ, η	= oblique coordinates
ρ	= mass density of plate material
ω	= angular frequency of plate
λ	$= (\omega a^2/\pi^2)(\rho h_0/D_{y_0})^{1/2}$, dimensionless frequency parameter

Introduction

THE parallelogramic flat plates are often used in structures of high-speed air vehicles. As no exact solutions exist to the governing differential equation for vibration analysis of skew plates of variable thickness and material properties, it requires attention of the designer to analyze such plates. For a typical configuration of skew plate simply supported along all the four edges, Seth[1] appears to be the first one to have given the exact solution. However, no other significant exact solutions exist for parallelogramic plates.

In recent years, several analyses of vibrations of isotropic as well as orthotropic skewed flat plates of uniform thickness have been presented by various authors. These investigations were based on the small-deflection theory of thin plates. Kaul and Cadambe[2] have used the Rayleigh's method to find the natural frequencies of isotropic skew plates of uniform thickness for different combinations of boundary conditions. The upper bound, as calculated by Rayleigh's method, and the lower bound of natural frequencies become inaccurate with decrease in the skew angle. Hasegawa[3] has given the lowest natural frequency for isotropic clamped plates, using the Ritz method. Hamada[4] has obtained the fundamental frequency of rhombic plates by employing the Lagrangian multiplier method. Conway and Farnham[5] have found the values of the first natural frequency of skew plates of uniform thickness, using the point-matching technique. Argyris and Buck[6] have solved the problem by the finite-element approach. Durvasula[7] has used the Galerkin method to get the first 6 to 8 natural frequencies of isotropic clamped plates of uniform thickness. Kumar and Pandalai[8] have reported the values of first six natural frequencies of orthotropic skew plates for three combinations of boundary conditions.

In this paper, the problem of free vibrations of isotropic as well as orthotropic parallelogramic plates with linearly varying thickness in one direction is investigated. The analysis is based on assuming a simple polynomial representation for the deflection surface in dimensionless oblique coordinates, satisfying the required polar symmetry and the boundary conditions of skew plate, and then employing the Galerkin's variational method. Numerical results for the parallelogramic plates with different combinations of aspect ratio, skew angle and taper parameter are presented. The properties of orthotropic material correspond to those of Maple 5-ply wood.

Derivation of the Differential Equation

Based on the small-deflection classical plate theory, the partial differential equation that governs the vibrations of skew plate with variable thickness is derived. The plate is considered to be made of orthotropic material. The constants that belong to orthotropic plate refer to the orthogonal system of axes. For the isotropic material, Poisson's ratio is taken to be 0.3.

Force Equilibrium Equation in z Direction

With reference to Fig. 1 and assuming the deflections and slopes to be small, summing the forces in the z direction yields

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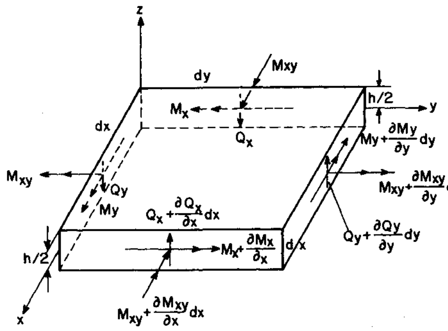

Fig. 1 Forces and moments acting on the plate element.

$$(\partial Q_x/\partial x)\,dx\,dy+(\partial Q_y/\partial y)\,dy\,dx = \rho h\,dx\,dy\,\partial^2 W/\partial t^2 \qquad (1)$$

Moment Equilibrium Equations

In summing moments about x and y axes, the equations reduce to[9]

$$\begin{aligned} Q_x-\partial M_x/\partial x-\partial M_{xy}/\partial y &= 0\\ Q_y-\partial M_{xy}/\partial x-\partial M_y/\partial y &= 0 \end{aligned} \qquad (2)$$

The moment equilibrium equation about z axis is identically satisfied. For orthotropic plate, moment resultants are given by

$$\begin{aligned} M_x &= -D_x(W_{,xx}+\nu_{yx}W_{,yy})\\ M_y &= -D_y(W_{,yy}+\nu_{xy}W_{,xx})\\ M_{xy} &= -2D_{xy}W_{,xy} \end{aligned} \qquad (3)$$

where subscripts after the comma refer to partial differentiation with respect to the independent variable(s). Also,

$$\begin{aligned} D_x &= E_x h^3/12(1-\nu_{xy}\nu_{yx})\\ D_y &= E_y h^3/12(1-\nu_{xy}\nu_{yx})\\ D_{xy} &= G_{xy}h^3/12 \end{aligned} \qquad (4)$$

Dividing throughout Eq. (1) by the elemental area $dx\,dy$ and using Eqs. (2) and (3), we obtain

$$(\partial^2/\partial x^2)[D_x(W_{,xx}+\nu_{yx}W_{,yy})]+(\partial^2/\partial y^2)[D_y(W_{,yy}+\nu_{xy}W_{,xx})]+4(\partial^2/\partial x\,\partial y)[D_{xy}W_{,xy}]+\rho h W_{,tt} = 0 \qquad (5)$$

with the assumption that $W_{,xy} = W_{,yx}$.

Assuming the harmonic solution

$$W(x,y,t) = w(x,y)\,e^{i\omega t} \qquad (6)$$

Eq. (5) takes the form

$$(\partial^2/\partial x^2)[D_x(w_{,xx}+\nu_{yx}w_{,yy})]+(\partial^2/\partial y^2)[D_y(w_{,yy}+\nu_{xy}w_{,xx})]+4(\partial^2/\partial x\,\partial y)(D_{xy}w_{,xy})-\rho h\omega^2 w = 0 \qquad (7)$$

Now, the linear thickness variation of the plate is assumed to be of the form

$$h = h_0(1+\alpha y/b) \qquad (8)$$

α and h_0 being constants.

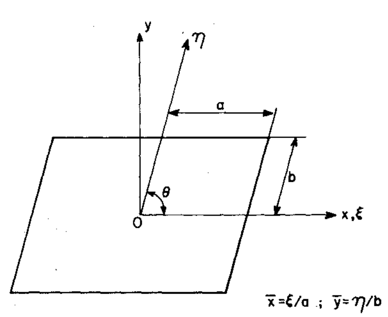

Fig. 2 Coordinates of the plate.

Using relation (8) in Eq. (4), we have

$$\begin{aligned} D_x &= D_{x_0}(1+\alpha y/b)^3\\ D_y &= D_{y_0}(1+\alpha y/b)^3\\ D_{xy} &= D_{x_0y_0}(1+\alpha y/b)^3 \end{aligned} \qquad (9)$$

where D_{x_0}, D_{y_0}, and $D_{x_0y_0}$ are constants. Substituting Eqs. (8) and (9) in Eq. (7) leads to

$$\frac{\partial^2}{\partial x^2}\left[D_{x_0}\left(1+\alpha\frac{y}{b}\right)^3(w_{,xx}+\nu_{yx}w_{,yy})\right]+\frac{\partial^2}{\partial y^2}\left[D_{y_0}\left(1+\alpha\frac{y}{b}\right)^3(w_{,yy}+\nu_{xy}w_{,xx})\right]+4\frac{\partial^2}{\partial x\,\partial y}\left[D_{x_0y_0}\left(1+\alpha\frac{y}{b}\right)^3(w_{,xy})\right]-\rho h_0\omega^2\left(1+\alpha\frac{y}{b}\right)w = 0 \qquad (10)$$

Simplifying reduces Eq. (10) to the following:

$$(1+\alpha y/b)^3[D_{x_0}(w_{,xxxx}+\nu_{yx}w_{,xxyy})+D_{y_0}(w_{,yyyy}+\nu_{xy}w_{,xxyy})+4D_{x_0y_0}w_{,xxyy}]+(6\alpha/b)(1+\alpha y/b)^2\times[D_{y_0}(w_{,yyy}+\nu_{xy}w_{,xxy})+2D_{x_0y_0}w_{,xxy}]+(6\alpha^2/b^2)D_{y_0}(1+\alpha y/b)(w_{,yy}+\nu_{xy}w_{,xx})-\rho h_0\omega^2\times(1+\alpha y/b)w = 0 \qquad (11)$$

Transformation to Oblique Coordinates

With reference to Fig. 2, the relations between rectangular and oblique coordinates are given by

$$x = \xi+\eta\cos\theta;\quad y = \eta\sin\theta \qquad (12)$$

Using the transformation relations as given by Eqs. (12) in Eq. (11) and multiplying throughout by a^4/D_{y_0}, and also with

$$(D_{x_0}/D_{y_0})^{1/2} = p_0;\quad (D_1+2D_{x_0y_0})/(D_{x_0}D_{y_0})^{1/2} = m_0$$
$$D_1/D_{y_0} = \nu_{xy};\quad D_1/D_{x_0} = \nu_{yx}$$

we obtain

$$\begin{aligned} &f_0^3[p_0^2w_{,\xi\xi\xi\xi}+2p_0m_0\times(\cot^2\theta w_{,\xi\xi\xi\xi}-2\cot\theta\,\mathrm{cosec}\,\theta w_{,\xi\xi\xi\eta}+\mathrm{cosec}^2\theta w_{,\xi\xi\eta\eta})+\\ &(\cot^4\theta w_{,\xi\xi\xi\xi}-4\cot^3\theta\,\mathrm{cosec}\,\theta w_{,\xi\xi\xi\eta}+6\cot^2\theta\,\mathrm{cosec}^2\theta w_{,\xi\xi\eta\eta}-4\cot\theta\,\mathrm{cosec}^3\theta w_{,\xi\eta\eta\eta}+\\ &\mathrm{cosec}^4\theta w_{,\eta\eta\eta\eta})]a^4+(6\alpha/b)f_0^2\times[(-\cot^3\theta w_{,\xi\xi\xi}+3\cot^2\theta\,\mathrm{cosec}\,\theta w_{,\xi\xi\eta}-\\ &3\cot\theta\,\mathrm{cosec}^2\theta w_{,\xi\eta\eta}+\mathrm{cosec}^3\theta w_{,\eta\eta\eta})+p_0m_0(-\cot\theta w_{,\xi\xi\xi}+\mathrm{cosec}\,\theta w_{,\xi\xi\eta})]a^4+\\ &(6\alpha^2/b^2)f_0[w_{,\xi\xi}(\nu_{xy}+\cot^2\theta)-2\cot\theta\,\mathrm{cosec}\,\theta w_{,\xi\eta}+\mathrm{cosec}^2\theta w_{,\eta\eta}]a^4 = \pi^4\lambda^2 f_0 w \end{aligned} \qquad (13)$$

where $f_0 = [1+\alpha(\eta/b)\sin\theta]$.

Making use of the nondimensional coordinates as given by

$$\bar{x} = \xi/a;\quad \bar{y} = \eta/b \qquad (14)$$

Eq. (13) takes the form

$$f_0^3[C_1w_{,\bar{x}\bar{x}\bar{x}\bar{x}}-C_2w_{,\bar{x}\bar{x}\bar{x}\bar{y}}+C_3w_{,\bar{x}\bar{x}\bar{y}\bar{y}}-C_4w_{,\bar{x}\bar{y}\bar{y}\bar{y}}+C_5w_{,\bar{y}\bar{y}\bar{y}\bar{y}}]-f_0^2[C_6w_{,\bar{x}\bar{x}\bar{x}}-C_7w_{,\bar{x}\bar{x}\bar{y}}+C_8w_{,\bar{x}\bar{y}\bar{y}}-C_9w_{,\bar{y}\bar{y}\bar{y}}]+f_0[C_{10}w_{,\bar{x}\bar{x}}-C_{11}w_{,\bar{x}\bar{y}}+C_{12}w_{,\bar{y}\bar{y}}] = \pi^4\lambda^2 f_0 w \qquad (15)$$

In the preceding,

$$\begin{aligned} C_1 &= p_0^2+2p_0m_0\cot^2\theta+\cot^4\theta\\ C_2 &= \gamma(4p_0m_0\cot\theta\,\mathrm{cosec}\,\theta+4\cot^3\theta\,\mathrm{cosec}\,\theta)\\ C_3 &= \gamma^2(2p_0m_0\,\mathrm{cosec}^2\theta+6\cot^2\theta\,\mathrm{cosec}^2\theta)\\ C_4 &= \gamma^3(4\cot\theta\,\mathrm{cosec}^3\theta)\\ C_5 &= \gamma^4\,\mathrm{cosec}^4\theta\\ C_6 &= \gamma(p_0m_0+\cot^2\theta)6\alpha\cot\theta\\ C_7 &= \gamma^2(p_0m_0+3\cot^2\theta)6\alpha\,\mathrm{cosec}\,\theta\\ C_8 &= \gamma^3 18\alpha\cot\theta\,\mathrm{cosec}^2\theta\\ C_9 &= \gamma^4 6\alpha\,\mathrm{cosec}^3\theta \end{aligned}$$

$$C_{10} = \gamma^2(\nu_{xy}+\cot^2\theta)6\alpha^2$$
$$C_{11} = \gamma^3 12\alpha^2 \cot\theta \operatorname{cosec}\theta$$
$$C_{12} = \gamma^4 6\alpha^2 \operatorname{cosec}^2\theta$$

Analysis

The problem of free vibrations of thin orthotropic parallelogramic plates of variable thickness is governed by the partial differential equation (15).

For the skew plate clamped on all the four edges, the boundary conditions are defined as

$$w = 0 \quad \text{and} \quad w_{,n} = 0 \quad \text{on all the edges} \tag{16}$$

In the above, n represents the direction of outward normal to the edge.

The boundaries of the plate in the dimensionless oblique coordinates (Fig. 2) are defined by

$$\bar{x} = \pm 1; \quad \bar{y} = \pm 1 \tag{17}$$

To satisfy the required conditions on the boundaries of the plate, a deflection function is chosen in the form

$$w(\bar{x}, \bar{y}) = (1-\bar{x}^2)^2(1-\bar{y}^2)^2 \sum_{m=0,1}^{\infty} \sum_{n=0,1}^{\infty} A_{mn}\bar{x}^m\bar{y}^n \tag{18}$$

where A_{mn} is the unknown coefficient.

On substituting w, as defined in Eq. (18), into Eq. (15), the equation reduces to the form

$$F_2(\bar{x}, \bar{y}) - \pi^4\lambda^2 F_1(\bar{x}, \bar{y}) = 0 \tag{19}$$

As in this case only a few parameters are considered for the computational purpose, the left-hand side of Eq. (19) represents an error function. The total work done by this error function during an incremental virtual displacement δw must be identically equal to zero.

Thus, we have

$$\int_{-1}^{+1}\int_{-1}^{+1} [F_2(\bar{x}, \bar{y}) - \pi^4\lambda^2 F_1(\bar{x}, \bar{y})]\,\delta w\, d\bar{x}\, d\bar{y} = 0 \tag{20}$$

The variation δw is expressed as

$$\delta w = \frac{\partial w}{\partial A_{00}}\delta A_{00} + \frac{\partial w}{\partial A_{01}}\delta A_{01} + \cdots + \frac{\partial w}{\partial A_{ik}}\delta A_{ik} + \cdots \tag{21}$$

Applying Galerkin's variational method, the resulting equation takes the form

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}[D_{mn}{}^{(ik)} - \pi^4\lambda^2 B_{mn}{}^{(ik)}] = 0 \tag{22}$$

$$\text{for } i = 0, 1, 2, \ldots; \quad \text{and} \quad k = 0, 1, 2, \ldots$$

Equation (22) represents an infinite set of linear, simultaneous, homogeneous equations in the infinite undetermined parameters, namely, A_{00}, $A_{01}, \ldots$. For computational task, only a definite number of terms in double series are considered. To examine the convergence, different number of terms have been used in the series for typical configuration of the skew plate. The characteristic equation is obtained by setting the determinant of the coefficients of A_{mn} in the matrix equation (22) equal to zero.

Numerical Computations

Numerical work has been carried out to compute the first few natural frequencies of parallelogramic plates with variable thickness for various combinations of aspect ratio, skew angle and taper parameter. Natural frequencies for both isotropic and orthotropic plates have been computed. As each one of the equations in Eq. (22) in which $(m+n)$ is even (odd) contains only those coefficients A_{mn} in which $(i+k)$ is also even (odd), it is easy to notice that the matrix equation (22) splits into two sets, the even set and the odd set, for $(m+n)$, $(i+k)$ even and $(m+n)$, $(i+k)$ odd, representing modes that are doubly symmetric or doubly antisymmetric and symmetric—antisymmetric or antisymmetric—symmetric, respectively. Using 16 terms in Eq. (8) by taking both m and n equal to 3, the matrices of order 8×8 have been solved to get the natural frequencies for the even set, using CDC 6400 Digital Computer. The even set includes the fundamental frequency of the plate configurations considered herein. Convergence study has been made by considering different number of terms in the double series for typical configuration of the plate.

Table 1 Frequencies λ of isotropic clamped skew plates of variable thickness: $p_0 = m_0 = 1$; $\nu = 0.3$

γ	α	Mode	$\theta = 90°$	$\theta = 75°$	$\theta = 60°$
1	0.0	1	3.6465	3.8692	4.6728
			3.6467[a]	3.870[a]	4.675[a]
			3.647[b]	3.879[b]	4.678[b]
		2	10.969	11.152	12.266
			10.970[a]	11.12[a]	12.11[a]
	0.4	1	3.8796	4.1043	4.9136
		2	11.704	11.838	12.896
	0.8	1	4.4385	4.6733	5.5139
		2	13.471	13.447	14.402
1/2	0.0	1	2.4906	2.6571	3.2649
			2.4907[a]	2.657[a]	3.265[a]
			2.491[b]	2.659[b]	3.269[b]
		2	4.6234	4.8397	5.6668
			4.5390[a]	4.730[a]	5.437[a]
	0.4	1	2.5699	2.7373	3.3467
		2	5.0710	5.2795	6.0921
	0.8	1	2.7632	2.9340	3.5538
		2	6.2374	6.4215	7.1983

[a] Reference 7.
[b] Reference 3.

Results and Discussion

Numerical results for the fundamental as well as the next higher natural frequency have been presented for isotropic and orthotropic plates with aspect ratios equal to 1 and $\frac{1}{2}$, skew angles equal to 90°, 75°, and 60°, and taper parameters equal to 0, 0.4, and 0.8. The results are presented in terms of the dimensionless frequency parameter λ. The natural frequencies are reported in the ascending order of their magnitude along with the mode number that is labeled accordingly. Table 1 gives the values of λ for isotropic clamped skew plates with some finite

Table 2 Frequencies λ of orthotropic clamped skew plates of variable thickness: $p_0 = 1.7664$; $m_0 = 0.3668$; $\nu_{xy} = 0.1206$

γ	α	Mode	$\theta = 90°$	$\theta = 75°$	$\theta = 60°$
1	0.0	1	4.8125	4.9889	5.6585
			4.8125[a]	4.9891[a]	5.6590[a]
		2	13.739	13.750	15.012
			13.489[a]	13.516[a]	14.747[a]
	0.4	1	5.0249	5.2043	5.8823
		2	14.558	14.677	15.856
	0.8	1	5.5531	5.7418	6.4499
		2	16.568	16.770	17.860
1/2	0.0	1	4.1041	4.1900	4.5448
			4.1041[a]	4.1900[a]	4.5449[a]
		2	5.5042	5.7656	6.7069
			5.4249[a]	5.6710[a]	6.5429[a]
	0.4	1	4.2004	4.2859	4.6391
		2	6.0026	6.2567	7.1790
	0.8	1	4.4288	4.5151	4.8711
		2	7.3258	7.5658	8.4506

[a] Reference 8.

Table 3 Convergence study for isotropic clamped skew plate: $p_0 = m_0 = 1$; $\nu = 0.3$; $\alpha = 0$; $\gamma = 0.5$; $\theta = 60°$

Order of Matrix	Mode 1	2	3	4
4×4	3.2796[a]	...	9.8631[a]	...
6×6	3.2705	6.1343	9.5833	...
8×8	3.2649	5.6668	9.5446	13.877
12×12	3.2648	5.6664	9.5036	13.759

[a] Figures in the Table represent the frequencies λ.

taper and with no taper at all. The comparison of frequencies in the case of plates with uniform thickness is also shown therein. Table 1 contains the results for various new configurations of skew plates of variable thickness. It may be seen that the authors' results are probably more accurate than that of at least some of the other investigators for the fundamental frequency of the skew plates with uniform thickness. The results of numerical calculation for the orthotropic plates are presented in Table 2 for two aspect ratios, three skew angles and three taper parameters. For the orthotropic plates, the properties of material correspond to those of Maple 5-ply wood. The study of convergence has been made for the isotropic plate with $\gamma = \frac{1}{2}$ and $\theta = 60°$, using each time different number of terms in the series expansion and solving different order matrix for the eigenvalues. Table 3 gives the results of this study. The convergence of double series used in representing the deflection surface of plate is satisfactory. It is also noticed that the deflection function, when assumed in such a representative form, is much more rapidly convergent in comparison with that based on characteristic functions of a clamped-clamped uniform beam[7,8] for the uniform plate vibration problems. It is because only 16 terms are required to get better results with the deflection function assumed herein as against the 36-term solution of the vibration problem making use of the beam eigenfunctions in case of at least the fundamental frequency of uniform thickness skew plates. Table 3 shows that 16 terms, using m and n each equal to 3, are sufficient to obtain reasonably good estimates of the natural frequencies up to the first two modes for plates with skew angle not below 60°. However, 16 terms may not be sufficient to obtain good results for any skew angles. The accuracy of results presented for plates with different configurations and material properties can be seen to be reasonable for skew angle as low as 60°. However, the results of calculation of the natural frequencies associated with higher modes of plates at low angles of skew are only rough estimates as a consequence of which these are not included here. The effect of skew angle on fundamental frequency of these plates, whether isotropic or orthotropic, appear to be practically the same. The frequency values for the fundamental mode of plates with different skew angles but with same aspect ratios and same taper parameters differ almost by a constant quantity. In other words, the effect of skew angle on the lowest natural frequency of plates with same aspect ratios and same taper parameters is almost the same. However, the frequency values continue to attain higher magnitudes for the orthotropic plates in comparison with that for the isotropic ones. This is probably attributable to the difference of material properties in the orthogonal directions.

Conclusions

The problem of free vibrations of thin, orthotropic, skewed flat plates is studied, using the Galerkin's variational method. The plates considered herein are clamped on all the four edges and are of uniform or variable thickness. A simple polynomial representation is chosen for the deflection surface which satisfies the required polar symmetry and boundary conditions of the plate. The numerical computations were carried out, in most of the cases, using 16 terms in the double series of deflection function. Numerical results for plates with various configurations are presented and the limited available data is inserted therein, wherever possible. The results for various new configurations of isotropic and orthotropic plates are also given. Convergence study has been made for a typical configuration of the plate. It appears that a 16-term solution is quite satisfactory for obtaining reasonably accurate results up to the first two even modes of plates clamped on all the edges. However, for higher modes at low skew angles it appears that more than 16 terms would probably be required to ensure better convergence. With the increase in aspect ratio, the natural frequency of plates increases. Also, with the orthotropic property of material, the frequency values tend to increase for the various configurations of plates.

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